LITERATURE CITED

1. A. M. Obukhov, "Kolmogorov flow and its laboratory simulation," Usp. Mat. Nauk, 38, No. 4 (1983).
2. A. M. Obukhov, Turbulence and the Dynamics of the Atmosphere [in Russian], Gidrometeoizdat, Leningrad (1988).
3. E. B. Gledzer, F. V. Dolzhanskii, and A. M. Obukhov, Systems of the Hydrodynamic Type and Their Application [in Russian], Nauka, Moscow (1981).
4. A. M. Batchaev, "An experimental investigation into supercritical self-oscillation regimes in Kolmogorov flows on a cylindrical surface," Izv. Akad. Nauk SSSR, FAO, 24, No. 8 (1988).
5. D. V. Lyubumov, G. F. Putin, and V. I. Chernotynskii, "Convective flows in a Haley—Shaw cell," Dokl. Akad. Nauk SSSR, 235, No. 3 (1977).
6. G. F. Putin and E. A. Tkacheva, "Experimental investigation into supercritical convective motions in a Haley—Shaw cell," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gazov., No. 1 (1979).
7. M. Z. Gak, "A laboratory study of self-oscillations in a four-vortex system," Izv. Akad. Nauk SSSR, FAO, 17, No. 2 (1981).
8. L. A. Pleshanova, "Oscillations in a four-vortex system," Izv. Akad. Nauk SSSR, FAO, 18, No. 4 (1989).
9. A. M. Batchaev, "Experimental investigation of self-oscillation characteristics in a four-vortex system with fluid layers of various thicknesses," Izv. Akad. Nauk SSSR, FAO, 25, No. 4 (1989).
10. F. V. Dolzhanskii, "The effect of external friction on the stability of plane-parallel flows in a uniform incompressible fluid," Izv. Akad. Nauk SSSR, FAO, 23, No. 4 (1987).
11. S. O. Lomadze, T. N. Nesterova, and A. S. Smirnov, "Automation of experimental projects at the IFA of the Academy of Sciences of the USSR," Preprint No. 2, Akad. Nauk SSSR, IFA, Moscow (1987).
12. L. D. Landau and E. M. Lifshits, Hydrodynamics [in Russian], Nauka, Moscow (1986).
13. K. F. Teodorchik, Self-oscillation Systems [in Russian], Gostekhizdat, Moscow (1948).

CALCULATING THE NONISOTHERMAL SEPARATION STREAMLINING OF A SPHERE

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UDC 532.516

Problems in technology frequently deal with the determination of resistance and heat-exchange factors for a solitary sphere, where its temperature is significantly different from that of the incoming flow of gas. In chemically reactive systems, moreover, the need arises for detailed knowledge of the fields of velocity and temperature in the flow about the particle.

A considerable number of studies (for example, [1-4]) has been devoted to the streamlining of a sphere by a uniform incompressible steady flow. These studies have enabled us to ascertain a detailed pattern of flow, coincident with experiment in such minute parameters as the angle of vortex separation and the length of the recirculation zone behind the trailing edge. Attempts have recently been made to calculate the nonisothermal problem [5], as well as the problem of the streamlining of a vaporized droplet in the case of small mass-exchange coefficients [6]. There exists a large quantity of work on the supersonic streamlining of a sphere at large Reynolds numbers Re_∞, a substantial portion of which is covered in [7, 8]. Hypersonic streamlining of a sphere at moderate values of Re_∞ is dealt with in [9], but these calculations are methodological in nature, owing to the fact that for description of the gas flow at the Reynolds and Mach numbers under consideration, when the Knudsen numbers $Kn = M_\infty/Re_\infty > 0.1$, and the Navier—Stokes equations, are, generally speaking, inapplicable. In studies dealing with the supersonic streamlining of a sphere, the authors have generally been interested in the characteristics of the flow in the forward part

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of the sphere, and these, basically, determine the coefficient of drag and heat exchange, as well as the parameters of the shock wave [10, 11]. However, as was suggested earlier, in the case of reacting combustion flows the characteristics of the flow behind the trailing edge of the sphere, particularly in the zone of flow separation, may exert a decisive effect on the parameters of the process.

We will examine the streamlining of a solitary spherical particle with a steady flow of gas that is uniform at infinity (without shear) at $Re_\infty \le 200$ and $0 \le M_\infty \le 2$. In the presence of vaporization the thermohysical parameters of the vapor are assumed to be identical to the parameters of the approaching flow. A two-dimensional axisymmetric formulation of the problem is assumed. The Navier–Stokes equations are used to describe the behavior of the flow. We choose a spherical coordinate system whose origin is at the center of the sphere.

At some distance from the sphere, where $r = R_\infty$ and $0 \le \theta \le \pi/2$, the following values are given for the unperturbed flow, namely:

$$v_r = -\cos\theta,\ v_\theta = \sin\theta,\ T = 1,\ p = 0.$$

Behind the trailing edge of the sphere ($\pi/2 < \theta \le \pi$) we have the following soft boundary conditions:

$$\partial v_r/\partial r = \partial v_\theta/\partial r = \partial T/\partial r = \partial p/\partial r = 0.$$

On the $\theta = 0$ and $\theta = \pi$ axes the following conditions of axial symmetry are specified:

$$v_\theta = \partial v_r/\partial\theta = \partial T/\partial\theta = \partial p/\partial\theta = 0.$$

With small Knudsen numbers Kn at the surface of the sphere we employ conventional conditions of adhesion and temperature constancy ($v_\theta = 0$, $T = T_w$), while in the case of rather large Kn (Kn ≥ 0.015), we have boundary slippage conditions and a temperature discontinuity [12]:

$$\begin{aligned} v_\theta &= 2.862\frac{M_\infty}{Re_\infty}\sqrt{\frac{\gamma}{T}}\frac{\mu}{\rho}\frac{\partial v_\theta}{\partial r}, \\ T - T_w &= 1.47\frac{M_\infty}{Pe_\infty}\frac{\gamma}{\gamma-1}\sqrt{\frac{\gamma}{T}}\frac{\lambda}{\rho}\frac{\partial T}{\partial r}. \end{aligned} \tag{1}$$

If the temperature of the sphere is equal to the boiling point, the no-flow condition $v_r = 0$ is replaced by the expression

$$v_r = \frac{2B}{Pe_\infty\alpha}\frac{\lambda}{\rho}\frac{\partial T}{\partial r}$$

($B = C_{p\infty}T_\infty\alpha/L$ is the coefficient of mass exchange and L is the specific heat of vaporization, $\alpha = 1 - T_w$), characterizing the relationship between the quantity of vaporized material and the flow of heat into the sphere.

We adopt the equation of state for an ideal gas. The thermophysical properties of the gas are assumed to be functions of temperature and are calculated with the aid of the ASTRA-3 computational program for states in thermodynamic equilibrium [13].

We used the generally accepted notation. All variables have been made dimensionless in terms of their values in the unperturbed flow. The exception involves the relative pressure p and the coordinate r, referred respectively to $\rho_\infty V_\infty^2$ and R_s the radius of the sphere. The Reynolds number is determined from the diameter of the sphere: $Re_\infty = 2\rho_\infty V_\infty R_s/\mu_\infty$. The quantity $Re_M = 2Re_\infty/(1 + \mu_w)$ will also be used later on. The subscripts ∞ and w pertain to values of the parameters in the unperturbed flow and at the surface of the sphere.

To solve the formulated problem we employ the method of division by physical processes and spatial direction. The steady solution is worked out by an established method. The difference scheme is constructed on the basis of implicit methods such as those discussed in [7, 8]. In the case of an incompressible flow, it is similar to the scheme taken from [7, p. 145]. The integration region is converted to a unit square by means of the transformations $n = \log_{R_\infty} r$, $s = \theta/\pi$, in which a uniform grid is constructed. The convective terms here are replaced by differences counter to the flow, with second-order accuracy.

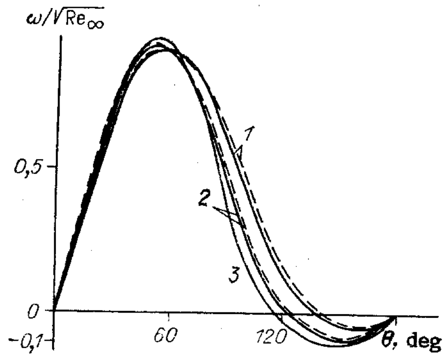

Fig. 1

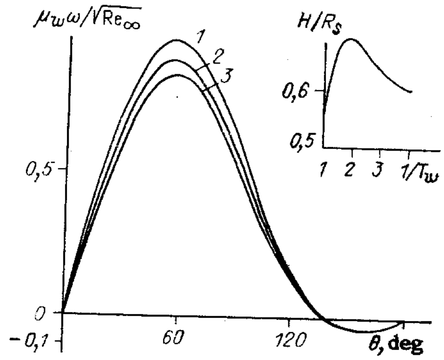

Fig. 2

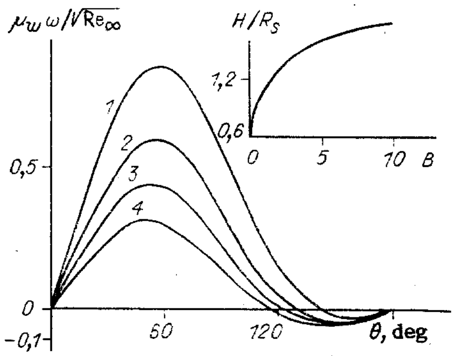

Fig. 3

The diffusion terms and expressions of the type grad P and div **v** are approximated by central differences. It must be noted that owing to the nonuniformity of the grid with respect to the variable r there exists a schematic viscosity ε defined by the expression $\varepsilon = \rho|v_r|r\Delta r_{min}^2$. All of the grid variables with the exception of pressure are determined at the nodes of the main grid. The pressure is calculated at the nodes of an auxiliary grid, displaced relative to the main grid through a half-step, in both directions. The chosen pattern makes it possible to eliminate the need for determining, in the course of the calculations, the vorticity quantities and those of the heat flow at the surface of the sphere.

The computational algorithm in the single time step $\tau = t_{k+1} - t_k$ is broken down into several stages. With the exception of the last stage, the difference scheme is constructed analogously [8], i.e., with use of the method of dividing into spatial directions the preliminary values of derivatives are calculated over time for the flow density and the temperature, namely: $(\rho\mathbf{v})^*$ and T^*. The convective and dissipative terms in the equations of motion and energy are accounted for implicitly. The continuity equation, which in terms of the spatial variables is not separated, in order to avoid additional limitations on the interval over time with M, close to zero, is solved in the concluding stage. The difference scheme here assumes the form:

$$\rho' + \operatorname{div}(\rho\mathbf{v})^k + \tau \operatorname{div}(\rho\mathbf{v})' = 0; \tag{2}$$

$$(\rho\mathbf{v})' - (\rho\mathbf{v})^* = -\tau \operatorname{grad} p';$$

$$T' - T^* = -\tau(\gamma - 1)T^k \operatorname{div}\left[\frac{(\rho\mathbf{v})'}{\rho^k}\right], \quad \rho' T^k + \rho^k T' = \gamma M_\infty^2 p'. \tag{3}$$

The prime for the variables denotes the difference analog of its derivative with respect to time. The equation for p', derived after transformation, has the form

$$\frac{\gamma M_\infty^2 p'}{T^k \tau^2} - \operatorname{div}(\operatorname{grad} p') - (\gamma - 1)\rho^k \operatorname{div}\left(\frac{1}{\rho^k}\operatorname{grad} p'\right) =$$

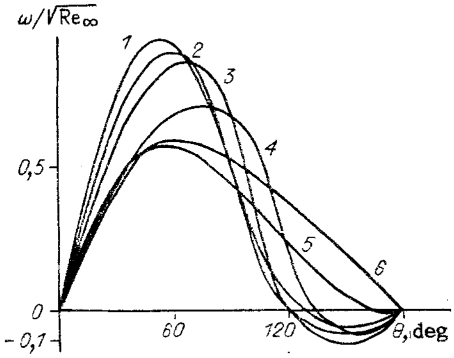

Fig. 4

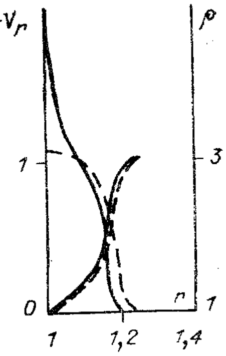

Fig. 5

$$= \frac{-\operatorname{div}\left[(\rho \mathbf{v})^k + \tau (\rho \mathbf{v})^*\right] - \tau(\gamma - 1)\rho^k \operatorname{div}\left[(\rho \mathbf{v})^*/\rho^k\right] + \frac{\rho^k}{T^k} T^*}{\tau^2}. \quad (4)$$

Construction of the difference equation for p' near the solid surface is accomplished in analogy with the methods from [7], utilizing a checkerboard array; however, this pattern imposes limitations on the near-boundary space intervals $\Delta r/r \leq \sqrt{2}\Delta\theta$. For the given problem this does not prove overly burdensome, owing to the need of consolidating the grid with respect to the coordinate r near the sphere in order to achieve an exact approximation of the gradients for the unknown quantities in this direction. Equation (4) is solved with the aid of the method of block symmetry relaxation, subsequent to which $(\rho v)'$ and T' are determined from (2) and (3).

Calculations of the streamlining of a sphere by an incompressible fluid on grids of various dimensions and with variation of R_∞ were carried out for testing purposes. Removal of the outside boundary by more than 20 radii leads to no noticeable change in the flow parameters near the sphere. The number of points at the angle N_θ was varied from 31 to 61, while along the N_r radius this number changed from 61 to 151. The fraction of schematic viscosity in this case did not exceed 1% for all of the calculations. It developed that the 101 × 31 grid yields fully acceptable results for $Re_\infty = 200$. The calculations were carried out until the condition $|p'| < 0.001$ was satisfied throughout the entire computational region. Establishment was achieved within 80-300 time steps, depending on the initial approximation.

On the whole, the results from the calculation of streamlining with an incompressible fluid are in good agreement with known theoretical and experimental data [1-4, 14]. The distribution of vorticity $\omega = 1/r[\partial/\partial r(rv_\theta) - \partial v_r/\partial\theta]$ over the surface of the sphere when $M_\infty = 0$ and $T_w = 1$ in comparison with the data of [7] (indicated by the dashed line) are shown in Fig. 1 (1-3: Re_∞ = 40, 100, and 200). The separation angle derived from a numerical solution with $Re_\infty = 200$ is equal to 62.5°, which is also in good agreement with the data from [4].

Comparison of the calculated coefficient of heat exchange in the case of nonisothermal streamlining against the approximation expression $Nu = 2 + 0.6 \cdot Re_M^{1/2} Pr^{1/3}$, presented in [15], yields a difference of no more than 5%. Thus, agreement of the numerical solutions with the experimental and theoretical data allows us to hope for an adequate description of the flow in the region for which no experimental information is at hand.

Figure 2 shows the distribution of vorticity over the surface of the sphere for Re_∞ = 40, $M_\infty = 0$ at various differences in the temperatures T_w and for various relationships between the length H of the recirculation zone and the temperature drop (1-3: T_w = 0.5, 0.33, and 0.25). The angle of separation as a function of T_w is very weak and is not monotonic. Analogous nonmonotonicity, but expressed with considerably greater clarity, appears also in the relationship between the length of the vortex and the temperature difference. Selective verification of the effect of the position of the R_∞ boundary and the number of grid nodes on the length of the vortex led to no noticeable change in the results.

Figure 3 shows the distribution of vorticity over the surface of the sphere for Re_∞ = 40, $M_\infty = 0$, $T_w = 0.25$ and for various mass-exchange coefficients B (1-4: B = 0, 1, 3, and 10) and the relationship between H and B. As we can see, as B increases, we observe a substantial reduction in vortex intensity and a significant increase in the length of the vortex.

Figure 4 shows a graph for the distribution of vorticity when $Re_\infty = 100$, $T_W = 1$, and $0 \leq M_\infty \leq 2$ (1-6: $M_\infty = 0$, 0.7, 0.9, 1.1, 1.5, and 2.0). With an increase in M_∞, given a subsonic regime for the incoming flow, the angle of separation shifts upstream, and this is in agreement with the remarks made in [16]. With a supersonic flow regime we observe the reverse process of a reduction in the angle of separation as M_∞ increases. The contention in [16] to the effect that the angle of separation increases as M_∞ increases therefore apparently pertains only to the subsonic flow regime.

With $Re_\infty = 100$ and $M_\infty = 0.9$ we observe an extensive local supersonic zone; however, unlike the case of the streamlining of a sphere with a nonviscous gas, where the supersonic zone appears as early as $M_\infty = 0.6$ [9], in the case of viscous streamlining with $Re_\infty = 100$ and $M_\infty = 0.7$ no local supersonic zone is observed and this is obviously related to the dissipative effect.

The influence of the boundary conditions related to temperature in a supersonic flow regime, insofar as this pertains to the parameters of the shock wave, are probably not overly significant, and this follows from a comparison of the results from [10], in which the condition $\lambda T/\partial r = 0$ is specified for the surface of the sphere, said comparison conducted relative to the computational data obtained with the condition $T = T_W$. Figure 5 shows the distribution of the velocity v_r and of the density ρ on the axis $\theta = 0$ for $Re_\infty = 180$, $M_\infty = 2$. The dashed line shows the plot of the data from [10].

The considerable effect of the finiteness of Kn on the dimensions of the vortex and the point of separation is shown. Thus, the result of the solution for the problem with $Re_\infty = 100$ and $M_\infty = 1.5$ and the standard conditions of adhesion and constancy of temperature yields magnitudes for the vortex and the angle of separation larger approximately by a factor of 2 than those obtained in calculations with conditions (1). It should be noted that the calculations carried out for Kn = 0.015-0.02 with the conditions of adhesion leads to a considerable (≈15%) divergence from the experimental data in [17] with respect to the coefficient of resistance. However, introduction of boundary conditions (1) leads to agreement with the results from [17].

Thus, numerical investigation demonstrated the substantial effect of vaporization and of Kn on the parameters of the separation zone. We also noted a qualitative change in the behavior of the vortex as related to the streamlining regime, i.e., an increase in its dimensions as M_∞ increase from 0 to 1, and a sharp reduction in the vortex with a further increase in M_∞.

LITERATURE CITED

1. A. E. Hamielec, T. W. Hoffman, and L. L. Ross, "Numerical solution of the Navier—Stokes equations for flow past spheres," AIChE J., 13, No. 2 (1967).
2. Y. Rimon and S. I. Cheng, "Numerical solution of a uniform flow over the sphere at intermediate Rynolds numbers," Phys. Fluids, 12, No. 5 (1969).
3. V. N. Shepelenko, "The streamlining of a sphere with the flow of a viscous fluid at low Reynolds numbers," ChMMSS, 5, No. 3 (1974).
4. S. I. Cheng, "A critical review of a numerical solution of Navier—Stokes equations," Lect. Not. Phys., 41, 78 (1975).
5. N. N. Sayegh and W. H. Gauvin, "Numerical analysis of variable property heat transfer to a single sphere in high-temperature surroundings," AIChE J., 25, No. 4 (1979).
6. M. Renksizbulut and M. S. Joan, "Numerical investigation of droplet vaporization with a high-temperature flow," Trans. ASME, Series S, Heat Transfer, No. 2 (1983).
7. O. M. Belotserkovskii, Numerical Modeling in the Mechanics of Continuous Media [in Russian], Nauka, Moscow (1984).
8. V. M. Kovenya and N. N. Yanenko, The Division Method in the Problems of Gasdynamics [in Russian], Nauka, Novosibirsk (1981).
9. A. V. Babakov, Numerical Simulation of Certain Problems in Aerohydrodynamics, VTs Akad. Nauk SSSR, Moscow (1986).
10. V. M. Kovenya, "Numerical simulation of problems in aerodynamics, based on the division method," Dissertation, Physical-Methematical Sciences, ITPM SO Akad. Nauk SSSR, (01.02.05), Novosibirsk (1982).
11. Yu. P. Golovachev and A. S. Kanailova, "A numerical study of the effects of slippage and temperature discontinuity at the surface of a sphere streamlined with a supersonic flow," JEP, 53, No. 3 (1987).

12. M. N. Kogan, The Dynamics of Rarefied Gases [in Russian], Nauka, Moscow (1967).
13. V. G. Trusov, S. A. Bardak, V. P. Turov, et al.," An automated system of thermodynamic data and calculations of equilibrium states," in: Mathematical Methods of Chemical Thermodynamics [in Russian], Nauka, Novosibirsk (1982).
14. S. Taneda, "Studies of wake vortices, experimental investigation of wake behind a sphere at low Reynolds numbers," J. Phys. Soc. Jpn., 11, No. 10 (1956).
15. Y. S. Lee, Y. P. Chyon, and E. Pfender, "Particle dynamics and particle heat and mass transfer in thermal plasmas. Pt. II. Particle heat and mass transfer in thermal plasmas," Plasma Chem. Plasma Proc., 5, No. 4 (1985).
16. L. D. Landau and E. M. Lifshits, Hydrodynamics [in Russian], Nauka, Moscow (1986).
17. A. W. Baily and J. Hiatt, "Sphere drag coefficients for a broad range of Mach and Reynolds numbers," AIAA J., 10, No. 11 (1972).

DEVELOPING MODELS TO CALCULATE THE EXCHANGE OF HEAT UNDER CONDITIONS OF SUPERSONIC TURBULENT DETACHED FLOWS

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Research into the processes of heat exchange in various turbulent flows is of great theoretical and practical interest. Among the more complex and urgent problems of aerogasdynamics we can include, with considerable certainty, the study of turbulent detached flows [1]. At supersonic flow velocities the determination of heat-exchange intensity in the vicinity of the separation zones assumes particular importance [2]. With significant changes in the level of turbulence within the external flow, in the boundary layers at the walls, and in the detached intermixing layers [3], methods based on simple correlations of heat-exchange parameters with characteristic pressures, see, for example [4-6], are rather limited. The approach proposed in [7] that is based on the utilization of a model of a nonequilibrium boundary layer seems to be more promising, and in addition to the factors of compressibility, nonisothermicity, and others, which are dealt with in this method in addition to the factors considered within the framework of asymptotic theory [8], the influence of a change in the intensity of large-scale turbulence is also considered. The heat-exchange calculations conducted in [7] for the vicinity of a cavity are in good agreement with the experimental data and the development of such an approach can expediently be applied to other conditions. It is with this purpose in mind that we have conducted additional experimental studies into the quasi-two-dimensional separation of flow in the vicinity of inclined protrusions and recesses [9]. The chosen geometric configuration has enabled us to analyze the effect of sequential interaction between the turbulent boundary layer and the compression shock and rarefaction waves insofar as these related to the intensity of heat exchange. Resort to the extensive additional information derived for these situations in [3, 10, 11] on the basis of utilizing a complex of various diagnostic methods: visualization of the extreme streamlines, optical and pneumometric measurements of pressure and velocity fields, thermoanemometric measurements of the characteristics of turbulence, all of these have enabled us to refine flow structure and the characteristic physical properties in order to validate the computational model being developed here with respect to new conditions.

Heat-exchange measurements were conducted in a wind tunnel with an operating wind-stream diameter of 304 mm within an Eifel chamber at incident-flow Mach numbers of $M_1 = 2$, 3, and 4. The individual Reynolds numbers varied within a range of $Re_1 = (30\text{-}91)\cdot 10^6\ m^{-1}$, the deceleration pressure $p^* = 200\text{-}1540$ kPa, and the deceleration temperature $T^* = 255\text{-}270$ K.

The studied configurations formed an inclined step oriented counter to the flow (Fig. 1b) or an inclined recess, streamlined in the opposite direction, with a fixed height h = 6 mm and a face deflection angle of $\beta = 25°$. The distance from the leading edge of the plate to the apex of the compression angle in the case of the protruding step amounted to 141 mm, and to 150 mm in the case of the apex of the expansion angle on the model of the re-

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